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Coherence effects and neutrino pair bremsstrahlung in neutron stars

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Abstract

We calculate the rate of energy radiation by bremsstrahlung of neutrino pairs by baryons in neutron stars employing a transport model where neutrinos couple to baryons with spectral width. The coherence effects, which are included by computing the self energies with fully dressed propagators, lead to the Landau–Pomeranchuk–Migdal suppression of the neutrino emission rates in the soft neutrino and high temperature limit. A microscopic evaluation of the bremsstrahlung rate by neutrons is carried out using the Brueckner theory of nuclear matter at finite temperatures.

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1. Introduction

Neutrino pair bremsstrahlung by baryons, $B_1 + B_2 \to B_1 + B_2 + \nu + \bar{\nu}$, is among the dominant processes by which neutron stars radiate away their energy during the first several thousand years of their evolution. The rates of these processes have been derived by Friman and Maxwell [1] and Voskresensky and Senatorov [2] employing the Fermi-liquid interaction combined with different prescriptions for pion exchange contribution to the nuclear matrix element.

The spectator in the reaction above is needed to insure that the process is allowed kinematically. [1] If the formation length of the neutrino radiation is of the order of the mean free path of a baryon, the role of the spectator is taken over by the medium because the baryon undergoes multiple scattering during the radiation. Then the reaction rate is subject to the Landau–Pomeranchuk–Migdal (LPM) suppression well-know from

[1] The neutrino pair bremsstrahlung via the reaction $B \to B + \nu + \bar{\nu}$ is inefficient as the energy conservation requires $\omega \sim q v_F$ where q is the momentum transfer and v_F is the baryon Fermi velocity, while for neutrinos in the time-like region $\omega \geq qc$. In more formal terms the imaginary part of the quasi-particle single-loop polarization function vanishes in the low-temperature limit.

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QED. At not too low temperatures the LPM suppression of the bremsstrahlung could be a factor in neutron stars, for the radiation frequency is of the order of temperature and could have a magnitude comparable to the baryon's collisional width.

The LPM effect has been naturally recovered in quantum many-body theory in attempts to overcome the limitations of the quasi-particle approximation which led to infrared-divergent results in the soft limit. Knoll and Voskresensky, Ref. [3], developed a general approach to the LPM effect using the Schwinger–Keldysh formulation of non-equilibrium field theory. The particle production rates in their approach are calculated from an expansion of the self-energies in terms of skeleton (or compact) diagrams with fully dressed particle propagators. Raffelt and Hannestad and Raffelt [4] have discussed in the supernova context the neutrino pair bremsstrahlung using the one-pion exchange interaction and including leading order corrections in particle width.

In this Letter we shall adopt the formalism of Ref. [3] to compute the rate of the neutrino pair bremsstrahlung from baryonic matter in neutron stars and supernovae, including the LPM effect. The spectral width of baryon propagators, which enters the rates of the bremsstrahlung, is derived from the finite temperature Brueckner theory of nuclear matter. Friman and Maxwell's [1] result is adopted as the reference for comparison.

The rates of neutrino pair bremsstrahlung in the Schwinger–Keldysh technique can be derived using the formalism of the optical theorem [5]. Here we follow a somewhat different path. It is known that the rates of the neutrino emission can be recovered from the Boltzmann equation for neutrinos [6]. For present purposes the Boltzmann equation, which deals only with on-shell particles, is not sufficient. We shall recover neutrino emission rates form the Kadanoff–Baym transport equation [7] which includes the collisional coupling of neutrinos to baryons with a finite spectral width. As the dense cores of neutron star develop, apart from neutrons and protons, a large equilibrium fraction of hyperons [10], our discussion shall refer generally to baryons. We treat neutrinos as relativistic massless Dirac fermions within the relativistic Kadanoff–Baym transport theory [8,9]. Our final expression relating the neutrino emissivity to the polarization of the medium agrees with the one derived by Voskresensky and Senatorov [5] from the optical theorem.

2. Formalism

We start from the Kadanoff–Baym transport equation for the neutrinos: [2]

$$\left\{\gamma^{\mu},\partial_{x\mu}S^{<}(q,x)\right\}_{+}=\sigma^{<}(q,x)S^{>}(q,x)-\sigma^{>}(q,x)S^{<}(q,x),\tag{1}$$

where $q\equiv(\boldsymbol{q},q_0)$ and x are the neutrino four momentum and the center-of-mass space-time coordinate, respectively, $S^{>,<}$ and $\sigma^{>,<}(q,x)$ are the neutrino propagators and collisional self-energies, which are real-time contour ordered with fixed-time arguments on the lower (upper) and upper (lower) branches of the Schwinger–Keldysh contour, respectively. Treating neutrinos on-mass-shell, the propagators can be expressed in terms of the non-equilibrium distribution functions (Kadanoff–Baym ansatz [7])

$$S^{<}(q,x)=\frac{i\pi \not{q}}{\omega_{\nu}(q)}\left[\delta\left(q_0-\omega_{\nu}(\boldsymbol{q})\right)f_{\nu}(q,x)-\delta\left(q_0+\omega_{\nu}(\boldsymbol{q})\right)\left(1-f_{\bar{\nu}}(-q,x)\right)\right];\tag{2}$$

the $S^{>}(q,x)$ propagator follows from Eq. (2) via the interchange $f_{\nu}\leftrightarrow(1-f_{\nu})$. The collision integrals contain neutrino scattering, absorption and emission contributions of which only the latter one is relevant for the cold neutron stars.

[2] Terms which do not contribute to the left-hand-side of the transport equation in the mean field approximation are dropped.

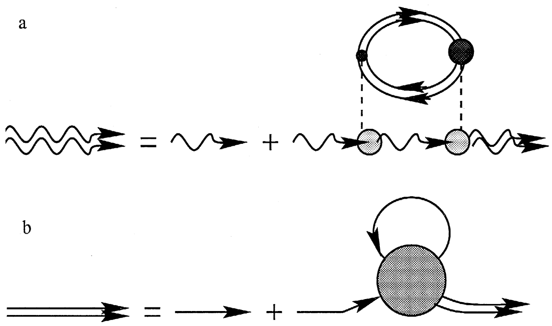

Fig. 1. Dyson equation for (a) neutrinos (wavy lines) and (b) baryons (solid lines). Double and single lines correspond to dressed and bare propagators, respectively, the dashed lines correspond to the W,Z boson exchange. The light dots denote the neutrino weak interaction vertex. The dark dot stands for the particle-hole vertex renormalization in the baryon loop. The shaded block in the lower panel is the thermodynamic T-matrix in the lowest order Brueckner theory.

The (anti-)neutrino emissivity (the power of the energy radiated per volume unit) is obtained by multiplying the left-hand-side of the transport equation by the neutrino energy and integrating over an element of the four dimensional phase-space:

$$\epsilon_{\nu\bar{\nu}} = \frac{d}{dt}\sum_f \int \frac{d^3q}{(2\pi)^3}\left[f_\nu(\boldsymbol{q}) + f_{\bar{\nu}}(\boldsymbol{q})\right]\omega_\nu(\boldsymbol{q}) = \sum_f \int \frac{d^3q}{(2\pi)^3}\left[I_\nu^{<,\mathrm{em}}(\boldsymbol{q}) - I_{\bar{\nu}}^{>,\mathrm{em}}(\boldsymbol{q})\right]\omega_\nu(\boldsymbol{q}), \tag{3}$$

where the reduced collision integral $I_\nu^{<,\mathrm{em}}(q)$ (em stands for emission) originates from energy integration over the range $[0,\infty]$ and corresponds to the neutrino branch of the spectrum, while $I_{\bar{\nu}}^{>,\mathrm{em}}(q)$ results from the integration over the range $[-\infty,0]$ and corresponds to the anti-neutrino branch of the spectrum; the sum is over the neutrino flavors. The leading order Feynman diagrams in the expansion of the self-energies $\sigma^{>,<}$ with respect to the weak neutrino-baryon coupling are shown in the Fig. 1a. The corresponding one-loop transport self-energies are read-off from the diagram

$$-i\sigma^{>,<}(q_1,x) = \int \frac{d^4q}{(2\pi)^4}\int \frac{d^4q_2}{(2\pi)^4}(2\pi)^4\delta^4(q_1-q_2-q)\, i\Gamma_q^{\mu}\, iS^{>,<}(q_2,x)\, i\Gamma_q^{\dagger\lambda}\, i\Pi_{\mu\lambda}^{>,<}(-q,x), \tag{4}$$

where $\Pi_{\mu\lambda}^{>,<}(q)$ are the Fourier transforms of the baryon polarization functions with time arguments fixed on the upper and lower branches of the contour. The baryon propagators in $\Pi_{\mu\lambda}^{>,<}(q)$ are fully dressed; the Fig. 1b displays the baryon propagator dressing due to the strong interaction within the lowest order Brueckner theory used in the numerical evaluation below. For small energy-momentum transfers involved in the problem, the weak interaction vertex, Γ_q^{μ}, can be replaced by the contact interaction: $\Gamma^{\mu} = \left(G/2\sqrt{2}\right)\gamma^{\mu}(1-\gamma^5)$, where G is the weak coupling constant.

Inserting the self-energies and the propagators in the left-hand-side of the transport Eq. (1) for the neutrino branch, we find, e.g., for the first term which is the gain part of the collision integral

$$\begin{aligned} I_\nu^{<}(q_1,x) = -i\int_0^{\infty}\frac{dq_{10}}{2\pi}\mathrm{Tr}\Bigg\{ &\int \frac{d^4q}{(2\pi)^4}\int \frac{d^4q_2}{(2\pi)^4}(2\pi)^4\delta^4(q_1-q_2-q)\,\Gamma^{\mu}\frac{\pi \not{q}_2}{\omega_\nu(q_2)} \\ &\times\left[\delta(q_{02}-\omega_\nu(\boldsymbol{q}_2))f_\nu(q_2,x) - \delta(q_{02}+\omega_\nu(\boldsymbol{q}_2))(1-f_{\bar{\nu}}(-q_2,x))\right] \\ &\times \Gamma^{\dagger\lambda}\frac{\pi \not{q}_1}{\omega_\nu(q_1)}\delta(q_{10}-\omega_\nu(\boldsymbol{q}_1))(1-f_\nu(q_1,x))\,\Pi_{\mu\lambda}^{>}(-q,x)\Bigg\}. \end{aligned} \tag{5}$$

One may identify various contributions to the collision integral: the terms $\propto (1-f_\nu)f_\nu$ and $\propto (1-f_\nu)(1-f_{\bar\nu})$ in the gain integral correspond to the scattering-in and emission processes, respectively. Similarly the loss term, $I_\nu^>$, which is obtained from Eq. (5) via the simultaneous interchange $f_{\nu,\bar\nu} \leftrightarrow (1-f_{\nu,\bar\nu})$ and replacement of $\Pi_{\mu\lambda}^>$ by $\Pi_{\mu\lambda}^<$, contains the terms $\propto f_{\nu,\bar\nu}(1-f_\nu)$ and $\propto f_\nu f_{\bar\nu}$ corresponding to scattering-out and absorption, respectively. As well known, e.g. Ref. [4], the neutrino mean free path exceeds the stellar radius and neutrinos leave the star without interactions except for several seconds after the neutron star birth. If neutrinos are untrapped, the only terms that contribute to the collision integral are the $\nu\bar\nu$ emission terms, one present in the neutrino gain integral (5) and the other in the anti-neutrino loss integral, $I_{\bar\nu}^>$ (which follows from Eq. (5) by replacing $\Pi_{\mu\lambda}^>$ by $\Pi_{\mu\lambda}^<$ and changing the integration limits).

Next we enforce the quasi-particle approximation in the collision integrals for neutrinos and anti-neutrinos by carrying out the q_{02} and q_{01} energy integrations. The gain and loss terms of the collision integrals are combined using the identities $\Pi_{\mu\lambda}^<(q) = \Pi_{\mu\lambda}^>(-q) = 2ig_B(q_0)\mathrm{Im}\,\Pi_{\mu\lambda}^R(q)$, where $g_B(q_0)$ is the Bose function and $\Pi_{\mu\lambda}^R(q)$ is the retarded component of the polarization function. Substituting the electroweak vertex in Eq. (3) and dropping the spatial argument, we find

$$\epsilon_{\nu\bar\nu} = -2\left(\frac{G}{2\sqrt{2}}\right)^2 \sum_f \int \frac{d^3q_2}{(2\pi)^3 2\omega_\nu(q_2)} \int \frac{d^3q_1}{(2\pi)^3 2\omega_\nu(q_1)} \int \frac{d^4q}{(2\pi)^4} (2\pi)^4 \delta^3(\boldsymbol{q}_2 + \boldsymbol{q}_1 - \boldsymbol{q})$$
$$\times \delta(\omega_\nu(\boldsymbol{q}_2) + \omega_\nu(\boldsymbol{q}_1) - q_0)[\omega_\nu(\boldsymbol{q}_2) + \omega_\nu(\boldsymbol{q}_1)]\, g_B(q_0)\, \Lambda^{\mu\lambda}(q_1,q_2)\mathrm{Im}\,\Pi_{\mu\lambda}^R(q), \qquad (6)$$

where $\Lambda^{\mu\lambda}(q_1,q_2) = \mathrm{Tr}[\gamma^\mu(1-\gamma^5)\not{q}_1\gamma^\lambda(1-\gamma^5)\not{q}_2]$is the trace of the neutrino currents. Here we made the approximation $f_\nu, f_{\bar\nu} \ll 1$ appropriate for untrapped neutrinos. The final expression for the emissivity, Eq. (6), recovered here from the transport equation, agrees with that derived from the non-equilibrium S-matrix expansion [5]. Analogous expressions for the scattering and absorption in equilibrium have been derived recently in Refs. [11,12].

The Lorentz structure of the baryon polarization function $\Pi_{\mu\lambda}(q)$ is given by the trace of the baryon transition currents

$$X_{\mu\lambda}(p,q) = \mathrm{Tr}\left[\gamma_\mu(c_V - c_A\gamma_5)(\not{p} + m)\gamma_\lambda(c_V - c_A\gamma_5)(\not{p} + \not{q} + m)\right].$$

The contraction of the neutrino trace with the non-relativistic reduction of $X_{\mu\lambda}$ simplifies the factor $\Lambda^{\mu\lambda}(q_1,q_2)\mathrm{Im}\,\Pi_{\mu\lambda}^R(q) = 8\omega_\nu(\boldsymbol{q}_1)\omega_\nu(\boldsymbol{q}_2)[c_V^2 L_V^R(\omega,\boldsymbol{q}) + 3c_A^2 L_A^R(\omega,\boldsymbol{q})]$ where the vector (L_V^R) and the axial vector (L_A^R) one-loop polarization functions differ by the particle-hole vertex renormalization. The vector part corresponds to the scalar particle-hole vertex; the axial-vector part corresponds to the spin exchange interaction vertex. These renormalizations can be carried out [5] in the Fermi-liquid theory, using the quasi-particle approximation, and amount to replacing the weak-coupling parameters c_V and c_A with the effective ones (explicit expressions are given in Ref. [5]). Eventually one is left with the factor $8\omega_\nu(\boldsymbol{q}_1)\omega_\nu(\boldsymbol{q}_2)(c_V^2 + 3c_A^2)L_0^R(\omega,\boldsymbol{q})$ where

$$L_0^R(\omega,\boldsymbol{q}) = 2\int \frac{d^3p\,d\varepsilon'\,d\varepsilon}{(2\pi)^5} \left\{ \frac{g^>(\boldsymbol{p},\varepsilon')g^<(\boldsymbol{p}+\boldsymbol{q},\varepsilon) - g^<(\boldsymbol{p},\varepsilon')g^>(\boldsymbol{p}+\boldsymbol{q},\varepsilon)}{\varepsilon' - \varepsilon + \omega + i\delta} \right\}, \qquad (7)$$

which is the driving term (and hence the subscript 0) in the resummation series for the particle-hole vertex; the parameters c_V and c_A here and below should be understood as the renormalized ones. The baryon propagators, $g^{>,<}(p)$, are related to the spectral function $a(p)$ and the Fermi distributions $f_N(p)$ via the Kadanoff–Baym ansatz for baryons [7]: $-ig^<(p) = a(p)f_N(p)$, $g^<(p) - g^>(p) = ia(p)$, which is exact in the equilibrium limit. The spectral function, in turn, is given by

$$a(\omega,\boldsymbol{p}) = -2\mathrm{Im}\left[\omega - \varepsilon_p + i\gamma(\omega,\boldsymbol{p})/2\right]^{-1}, \qquad (8)$$

where the quasi-particle energy $\varepsilon_p = \varepsilon_p^0 + \mathrm{Re}\,\Sigma^R(\omega,\boldsymbol{p})|_{\omega=\varepsilon_p}$, $\varepsilon_p^0 \equiv p^2/2m$ (i.e. the pole of the spectral function) and the inverse lifetime $\gamma(\omega,\boldsymbol{p}) = -2\mathrm{Im}\,\Sigma^R(\omega,\boldsymbol{p})$ (i.e. the width of the spectral function) are given in terms of the retarded baryon self-energy, $\Sigma^R(p)$; our approximation for the latter quantity is depicted in Fig. 1b. We note in passing that the imaginary part of the quasi-particle polarization function (the limit $\gamma(p)\to 0$) vanishes in the time-like region, $q_0^2 \geq \boldsymbol{q}^2$, in the low temperature limit. Eq. (7), therefore, is the leading order compact diagram contributing to the neutrino emission rate.

3. Results

Next we proceed to microscopic calculation of the self-energy and polarization function for the neutron matter, which is the dominant constituent of the dense stellar matter for densities $n \leq 2n_s$ ($n_s = 0.16$ fm^{-3}). The single particle energies and the width have been evaluated in the self-consistent lowest order Brueckner scheme at finite temperatures (Ref. [13] and references therein). This scheme requires a self-consistent solution of the coupled equations for the thermodynamic (retarded) T^R-matrix

$$T_{ll'}^{R\alpha}(p,p',P,\omega) = V_{ll'}^{\alpha}(p,p') + \frac{2}{\pi}\sum_{l''}\int dp''\, p''^2\, V_{ll''}^{\alpha}(p,p'')G_2^R(p'',P,\omega)T_{l''l'}^{R\alpha}(p'',p',P,\omega), \tag{9}$$

where α collectively denotes the quantum numbers (S,J,M) in a given partial wave, p and P are the

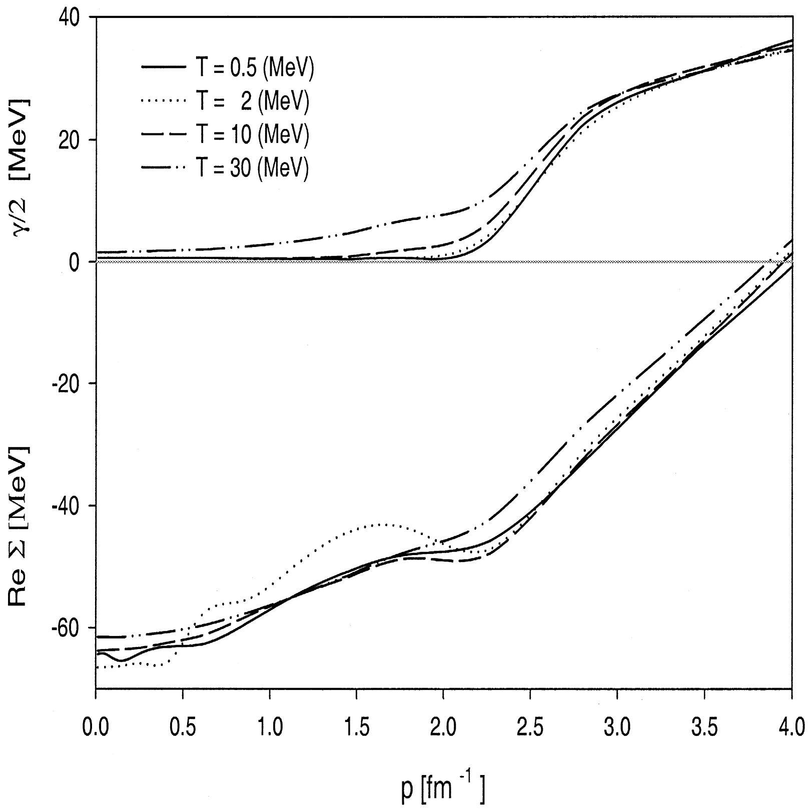

Fig. 2. The real part of the on-shell self-energy and the half-width as a function of particle momentum at the saturation density n_s for different temperatures; the zero temperature Fermi momentum is 1.7 fm^{-1}.

magnitudes of the relative and total momentum respectively, $V(p,p')$ is the bare nuclear interaction, G_2^R is the angle averaged two-particle propagator

$$G_2^R(p,P,\omega) = \int \frac{d\Omega}{4\pi} \frac{[1-f_N(\varepsilon(\boldsymbol{P}/2+\boldsymbol{p}))][1-f_N(\varepsilon(\boldsymbol{P}/2-\boldsymbol{p}))]}{\omega-\varepsilon(\boldsymbol{P}/2+\boldsymbol{p})-\varepsilon(\boldsymbol{P}/2-\boldsymbol{p})+i\delta}, \tag{10}$$

with $\varepsilon(p) = \varepsilon_p^0 + \mathrm{Re}\,\Sigma(\varepsilon_p,\boldsymbol{p})$ and the retarded self-energy

$$\Sigma^R(p,\omega) = \frac{1}{\pi}\sum_{l\alpha}(2J+1)\int dp'\, p'^2 T_{ll}^{R\alpha}(p,p';p,p';\omega+\varepsilon(p'))\, f_N(\varepsilon(p'));\tag{11}$$

here the lower case p is the magnitude of the proper momentum of a particle. The coupled Eqs. (9) and (11) are subject to normalization to the total density at a given temperature. Fig. 2 displays the real part of the on-shell self energy and the half width of the spectral function as a function of the particle momentum for several values of the temperature at the saturation density n_s. The numerical calculations were carried out using the Paris NN interaction keeping partial waves with $L \le 2$.

Let us next turn to the imaginary part of the polarization function, Eq. (7). One of the integrations can be removed by the delta function; further only the angular integral can be performed analytically [3]. We substitute the resulting expression in Eq. (6), carry out the neutrino phase-space integrals, and sum over their three flavors. The final result for the emissivity is

$$\epsilon_{\nu\bar{\nu}} = \frac{2G^2}{5(2\pi)^7}\left(c_V^2+3c_A^2\right) m^2\, T^7\, \mathscr{I}(T), \tag{12}$$

where T is the temperature,

$$\begin{aligned}\mathscr{I}(T) = \int_0^\infty dz\, z^5\, g_B(z)\int_{-\infty}^\infty dy[f_N(y)-f_N(y+z)]\int_0^\infty dx \frac{\chi(y,x)}{[y-\xi(x)]^2+\chi(y,x)^2/4}\\ \times\left\{\arctan\left[\frac{\xi(x)+\zeta(z)-y-z+2\sqrt{x\zeta(z)}}{\chi(y+z,x)/2}\right]\right.\\ \left.-\arctan\left[\frac{\xi(x)+\zeta(z)-y-z-2\sqrt{x\zeta(z)}}{\chi(y+z,x)/2}\right]\right\},\end{aligned} \tag{13}$$

with the quantities $\xi(x) = \varepsilon_p/T$, $\chi(y,x) = \gamma(\omega,p)/T$, $\zeta(y) = \varepsilon_q/T$ being functions of $x = \varepsilon_p^0/T$, $y = \omega/T$, $z = \omega'/T$; here ε_q is the recoil energy. Note that Eq. (12) contains only universal couplings [3] and constants as a prefactor, all the nuclear many-body effects are absorbed in the integral $\mathscr{I}(T)$, which is valid at arbitrary densities and temperatures (in particular does not rely on any type of low-temperature expansion). At low-temperature our result can be simplified by expanding the spectral function (8) to the leading order in γ. Performing the same steps leading to Eq. (12), we find in the first order γ

$$\epsilon_{\nu\bar{\nu}}^{(1)} = \frac{4G^2}{5(2\pi)^6}\left(c_V^2+3c_A^2\right) m^* p_F Z(p_F) T^7\int_0^\infty dz\, z^7\, g_B(z)\,\chi(z)\frac{\mathscr{P}}{(z-\zeta(z))^2}, \tag{14}$$

where p_F is the baryon Fermi momentum, m^* is the effective mass, and $Z(p_F)$ is the wave-function renormalization; $\mathscr{P}$ stands for the principal value integration. In this limit the width is essentially the reciprocal

[3] The values of parameters of weak interaction are $c_V = -1$, $c_A = -(F+D)$ for neutrons, $c_V = 1-4\sin^2\theta_W$, $c_A = F+D$ for protons, $c_V = -2+4\sin^2\theta_W$, $c_A = -F$ for Σ^- hyperons and zero for Λ hyperons; here $F = 0.477\pm0.012$, $D = 0.756\pm0.011$ and $\sin^2\theta_W = 0.23$, where θ_W is the Weinberg angle. As noted above, the particle-hole vertex corrections renormalize these values of c_V and c_A.

Table 1
The upper number for a density-temperature pair is the value of the triple integral $\mathscr{I}$, the lower number is the ratio of Eq. (15) to the result of Friman and Maxwell, Eq. (66a)

	T (MeV)					
	0.5	2	5	10	20	30
n_s	34.0	19.3	18.3	8.6	3.0	0.4
	2.46	0.35	0.13	3.1×10^{-2}	5×10^{-3}	5×10^{-4}
$2n_s$	33.0	18.8	17.64	5.49	3.84	2.12
	2.38	0.34	0.127	3.3×10^{-3}	7.0×10^{-3}	4.3×10^{-4}

of the quasi-particle life-time in a Fermi-liquid, hence $\gamma \propto T^2\left[1+(\omega/2\pi T)^2\right]$. Substituting this in Eq. (14) we recover the T^8 temperature dependence of the neutrino bremsstrahlung rate [1,2]. Note that according to Ref. [1] the matrix element for the Fermi transition vanishes in the limit $q \to 0$ in Born approximation. This cancellation occurs whenever the off-shell source particle propagation between the strong and weak interactions is odd under time-reversal, which is true in the quasi-particle approximation neglecting the recoil term. We can not make an accessment on this cancellation, as the quantum interference diagrams are not included so far in our treatment. Should this cancellation indeed emerge as a consequence of the current conservation requirements, [4] our numerical expression for the neutrino bremsstrahlung would overestimate the rate by a small factor $c_V^2/3c_A^2$.

A numerical estimate of the neutrino emissivity Eq. (12) for the case of the neutron bremsstrahlung gives

$$\epsilon_{\nu\bar{\nu}} = \left(7.56\times10^{18}\ \mathrm{erg\,cm^{-3}\,s^{-1}}\right)T_9^7\,\mathscr{I}(T). \tag{15}$$

where T_9 is the temperature in units 10^9 K. The triple integration in Eq. (13) is carried out numerically after the functions ξ,χ and ζ are derived from the microscopic theory. Table 1 shows the results of the calculations of the integral $\mathscr{I}(T)$ and compares the neutrino emissivity, Eq. (15), with the result of Friman and Maxwell [1].

4. Discussion

In this work we recovered the relation [Eq. (6)] between the neutrino emissivity and the polarization function of the nuclear medium from the Kadanoff–Baym transport theory. We derived the bremsstrahlung rates by computing the neutrino transport self-energies which are driven by neutrino coupling to baryons with spectral width. The diagrammatic expansion for the polarization function of the medium is constructed with fully dressed particle propagators, as originally suggested by Knoll and Voskresensky [3]. As a result the LPM effect, i.e. the influence of the multiple scattering during the radiation process, is included in the rate of neutrino emission. We have kept only the leading order term in the expansion given by the one-loop polarization; more complicated diagrams are not considered at this stage. Already at the leading order the rates are well behaved in the soft neutrino limit and exhibit LPM suppression when $\gamma/T \geq 1$. Since this ratio at low temperatures is proportional to T the LPM suppression increases with increasing temperature; in our case the typical scale for the onset of the LPM effect is $T \sim 5$ MeV. A comparison to the result of Friman and Maxwell [1] shows a strong suppression at relatively high temperatures and an enhancement by a factor of two in the low temperature limit. [5] We believe that the latter low temperature enhancement is mainly due to the differences in the treatment

[4] Note that the arguments of Ref. [1], perfectly valid in the limit discussed there, are not directly applicable to our discussion, as neither the particle width is an odd function of the frequency, nor the recoil term is neglected.

[5] Note that the extrapolation of the Friman and Maxwell's result to high temperatures is limited to the assumption that the quasi-particle picture can still be applied and that the quasi-particles are lying close to their Fermi surface (the expansion parameter being the ratio of the temperature over the Fermi energy).

of the nuclear interaction in the medium. For densities at and above the nuclear saturation density effects beyond those included in the Brueckner formalism, such as an enhancement of the strong interaction amplitude due to the in-medium modifications of the pion degrees of freedom, could compete with the LPM suppression, cf. [2].

Another feature of the expansion above is that the quantum interference terms appear only in higher order diagrams. The latter correspond e. g. to particle-hole insertions which transfer momentum between the particle and the hole Green's functions [3]. It is well known that at $T=0$ inclusion of these interference terms is required to fulfill the Ward–Takahashi identities [14]. At finite temperatures the interference corrections are reduced; an explicit evaluation at $T\neq 0$ has to be performed yet.

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